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THE GRADUATE SCHOOL

COLLEGE OF ENGINEERING

Optimization of Steering Geometry For Ultra-High-Mileage Vehicles

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Abstract:

With vehicles that seek to maximize fuel economy at any cost, for example the ultra-high-mileage vehicle competitions, even minor changes in vehicle setup can have noticeable effects on overall performance. This paper considers the fuel losses associated with the misalignment of the front steering geometry due to the typical trapezoidal Ackermann geometry setup. Starting with a standard front steering geometry, an algorithm is developed to calculate the optimal four-bar geometry for a vehicle and it is seen that such geometries can approach, but never reach, perfect Ackermann geometry on all individual tires simultaneously for low-speed turns. Using turning data collected from prior competitions, relationships between alignment angles and fuel economy were developed to predict decreases in fuel mileage due to tire scrub. The differences between optimized geometry and actual geometry were then used to predict the performance improvements of Penn State’s ultra-high-mileage entry in the Shell Eco-marathon Americas.

1. Introduction

Since the eighteenth century, the steering design of wheeled vehicles had been of interest. [1] Originally, to make a turn, horse-drawn carriages would pivot the whole front axle about a pivot point in the center (Fig. 1). This allowed for carriages to have a very small turning radius, but required that the front wheels be small enough to fit under the carriage without collision. Because wheels must maintain a minimum size to avoid entrapment into potholes, the only way such vehicles could negotiate a turn was to design a large cavity under the front of the carriage. This resulted in an upward shift of the vertical center of gravity. Additionally, the footprint of such a carriage in a turn transforms from a rectangular polygon for straight motion to a more triangular polygon for tight turns. Notably, the boundary of the support polygon on the outside of the turn moves inward. For situations where the carriage is negotiating a tight turn, the combination of high center of gravity and reduced support polygon greatly increases the likelihood of overturning the carriage.

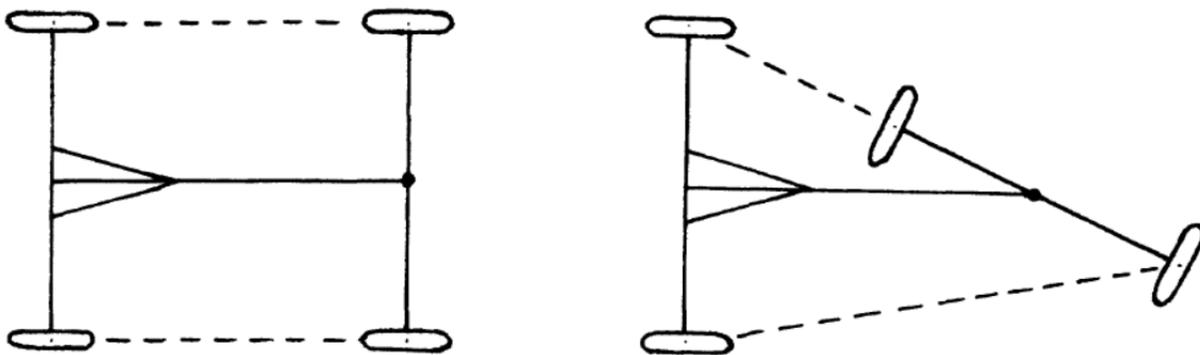


Figure 1. Example of straight travel and extreme turning of a carriage.

Image is Reproduced from D. King-Hele. [1]

Erasmus Darwin (father of Charles Darwin) and George Lankensperger have been credited for inventing the four-bar linkage to navigate turns in 1818. [1] This geometry today is known as Ackermann steering because it was the lawyer Rudolph Ackermann who patented it, patent no. 4212 (1818).

The basic geometrical setup of an Ackermann steering system is shown in Fig. 2. The early advantage of this method is that the support polygon of the vehicle remains largely unchanged and thus the stability of the carriage is not greatly affected at low speeds by the sharpness of the turn. But more importantly, the steering arms, because they are connected by a steering arm of shorter length than the axle, will inherently cause the inside wheels in a turn to steer more sharply than the outside wheels. The ideal geometry would be one where neither the inside or outside tire “scrubs” in a turn, e.g. is not aligned with the direction of vehicle travel. In the 1800’s, this type of misalignment between inner and outer tires was of large concern because roads were either dirt or gravel. If a wheel was not rolling tangentially to the overall path, the wheel drag increased significantly thus causing significant damage for roads with even moderate traffic. So bad were the effects of traffic on road wear that carriages started to implement larger wheels to increase ride comfort. [1]

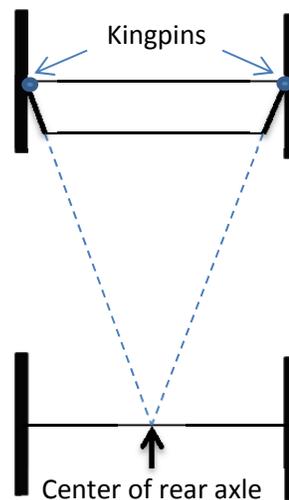


Figure 2. A general approximation of Ackermann geometry typically uses imaginary lines from the kingpin to the center of the rear axle.

Automobiles today still use a four-bar linkage system to navigate turns to avoid unnecessary tire scrub and hence damage to the tire. In order to resolve this issue each wheel needs to coincide on the same turn center. This would be pure Ackermann geometry. However, they do not implement pure Ackermann geometry and many ad-hoc design rules that claim to give pure Ackermann geometry are not justified by any analysis. [2] For most vehicles, this misalignment is so small that the typical driver rarely notices tire scrub during turns except for when they are turning the sharpest of corners.

However, for ultra-high efficiency vehicles, even small misalignments can be a significant portion of energy losses. Perfect Ackermann is crucial on every tire in order to decrease the amount of energy lost

due to misalignment of the tire with direction of travel. Because of the low speeds of such ultra-high efficiency vehicles, such misalignment has little benefit to the overall vehicle performance, and only produces waste heat.

2. Ideal Ackerman Geometry

In this section, the definition of the ideal Ackermann geometry is reviewed briefly, as this geometry will be used in later sections to determine deviations from ideal. Fig. 3 shows the relation between the radius of turn and the steering angles necessary for the inside and outside tires. For this paper, an “ideal” Ackermann geometry is defined as when the tire is perfectly tangent to a circle drawn from the center of the turn, e.g. when tires F and G are perpendicular to lines that intersect the center of the turn, as shown in Fig. 3.

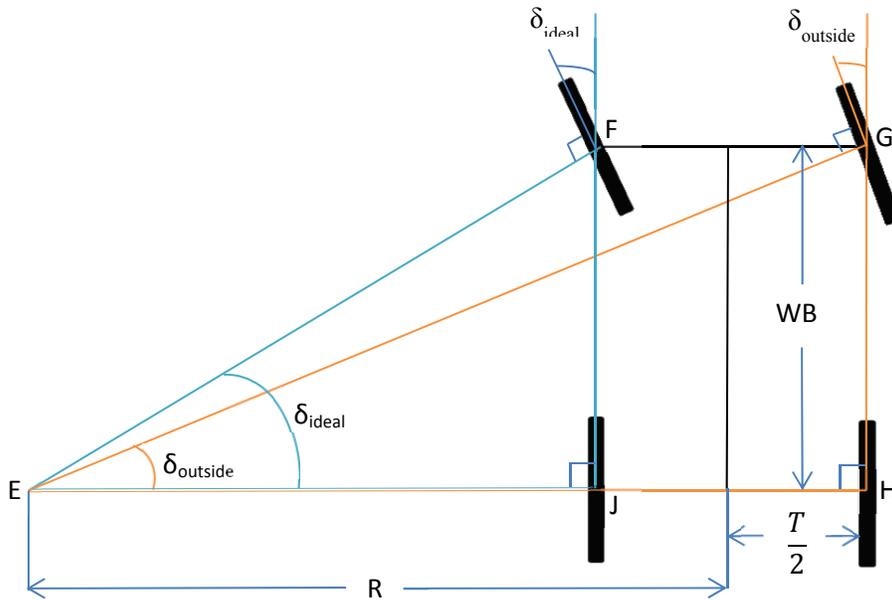


Figure 3. Relationship between $\delta_{outside}$ and δ_{inside} .

Looking at Fig. 3, one can see that two triangles are formed that can be used to derive the ideal steering angle. The ideal angle of the outside wheel is calculated using $\triangle EGH$,

$$\tan(\delta_{outside}) = \frac{WB}{(R + T/2)},$$

$$\xrightarrow{yields} R = \frac{WB}{\tan(\delta_{outside})} - \frac{T}{2}. \quad (1)$$

The ideal angle of the inside wheel is calculated using $\triangle EFJ$,

$$\tan(\delta_{ideal}) = \frac{WB}{(R - T/2)},$$

$$\xrightarrow{yields} \delta_{ideal} = \tan^{-1}\left(\frac{WB}{R - T/2}\right). \quad (2)$$

This ideal angle will be used in later sections to calculate the error between actual and ideal steering geometries.

3. Four-Bar Analysis to Derive the Optimal Ackerman Geometry

For very low speed turns like those seen in ultra-high-efficiency competitions, the most fuel efficient four-bar steering mechanism is one that minimizes any tire scrub. To derive this geometry, tire scrub forces must be derived from the steering geometry of the four-bar mechanism, the turn radius, and the tire parameters. To simplify the analysis that follows, we assume that the tires are in pure rolling; maneuvers are made at a low speed; wheels have equal static weights and no weight transfer occurs; and all maneuvers take place on a flat surface. For later numerical steps in this algorithm, the dimensions from the Penn State University entry are used. The outside wheel is assumed to be the driver of the steering mechanism.

Referring to Fig. 4, there are two dimensions that govern the four-bar arrangement: the track (T) and the wheelbase (WB). The Ackermann design uses a trapezium-style four-bar, one that is convenient to consider as a first approach because of symmetry, e.g. that the analysis is not dependent on the direction of the turn. A typical trapezoidal Ackermann geometry can be seen at the top of Fig. 4. The ground link of the four-bar, R_1 , is usually connected to a fixed frame and is hereafter assumed to have a fixed dimension, T. The lengths of R_2 and R_4 are assumed to be the same, but their angles will vary since they are dependent on R_3 and the Ackermann Angle (AA). We define the Ackermann Angle, AA, as the angle that is formed between the wheel and the steering arm, R_2 . Because this angle is mechanically set in most steering systems by the inset angle of the steering arm relative to the tire plane, it is hereafter assumed that the angle AA remains constant for a particular set-up. Consistent with the definition of low-speed steering, the radius of turn is defined as the distance from the turn center to the center of the rear axle.

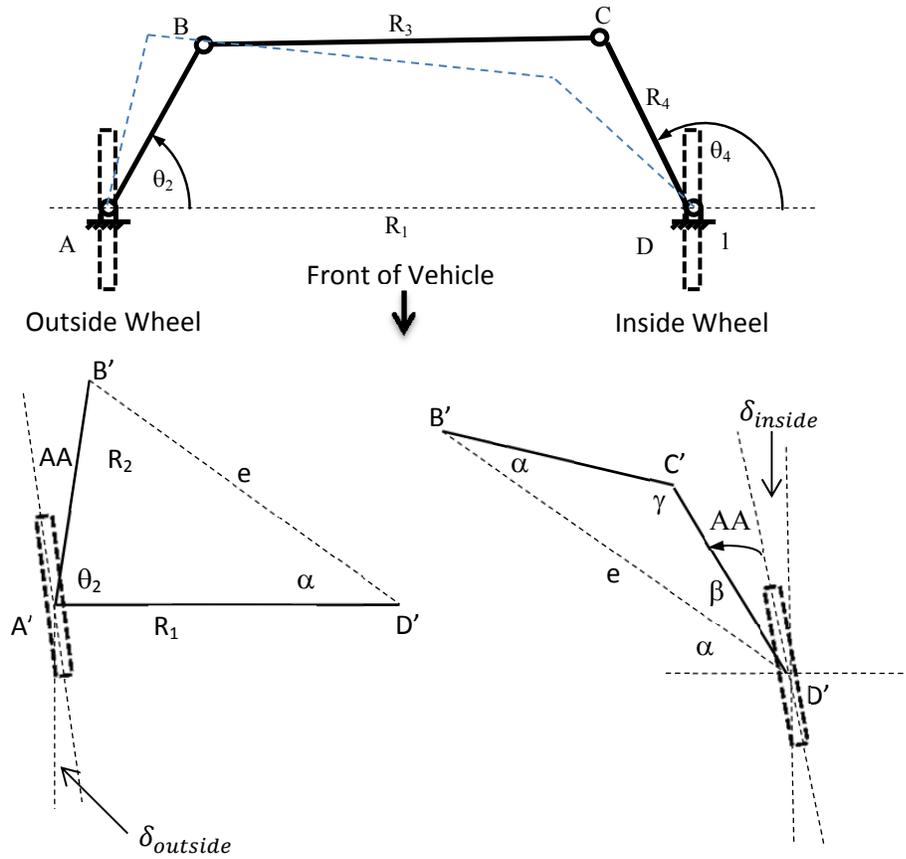


Figure 4. General representation of four-bar steering design for vehicles. [3]

Using the geometry defined previously for the special case when R_3 is parallel to R_1 , R_2 can be calculated by trigonometric relationships to obtain Eq.(1).

$$R_2 = \frac{1}{2} \frac{(T - R_3)}{\sin(AA)} \quad (3)$$

We now consider the ideal Ackermann geometry of each individual tire constrained by the trapezoidal mechanism above. During a turn, the four-bar can be modeled assuming that the link R_2 (connected to the outside tire) is the “driver” of the mechanism, and thus that the angle of the tire connected to R_2 is assumed to be set up to have perfect Ackerman geometry. In order to optimize the four-bar, we seek to solve for the angle of link R_4 in order to minimize scrub on the inside tire.

Using $\triangle ABD$, the length of triangle side, e , is given by

$$e^2 = R_1^2 + R_2^2 - 2R_1R_2 \cos \theta_2$$

$$\xrightarrow{\text{yields}} e = \sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos \theta_2} \quad (4)$$

The length, e , can then be used to obtain the angle α in ΔABD , which is used later.

$$\frac{\sin \theta_2}{e} = \frac{\sin \alpha}{R_2}$$

$$\xrightarrow{\text{yields}} \alpha = \sin^{-1} \left(\frac{R_2 \sin \theta_2}{e} \right) \quad (5)$$

We now relate the above geometries to ΔBCD in order to calculate the angle of the inside wheel. First, the length e is used to calculate γ ,

$$e^2 = R_3^2 + R_4^2 - 2R_3R_4 \cos \gamma$$

$$\xrightarrow{\text{yields}} \gamma = \cos^{-1} \left(\frac{R_3^2 + R_4^2 - e^2}{2R_3R_4} \right) \quad (6)$$

$$\frac{\sin \gamma}{e} = \frac{\sin \beta}{R_3}$$

$$\xrightarrow{\text{yields}} \beta = \sin^{-1} \left(\frac{R_3 \sin \gamma}{e} \right). \quad (7)$$

The steering angle of the inside wheel can now be found. When the wheels are straight,

$$\alpha + \beta + AA = 90^\circ, \quad (8)$$

and we also know that α and β change as a function of the driving link. Because the angle, AA , remains constant, the steering angle of the inside and outside wheel can now be found:

$$\delta_{inside} = |\alpha + \beta + AA - 90^\circ| \quad (9)$$

$$\delta_{outside} = \theta_2 + AA - 90^\circ. \quad (10)$$

4. Determination of Ackermann error

Using the previous equations, one can find the orientation error between the ideal angle of the inside tire, and the actual angle of the inside tire, for a variety of steering conditions that define the angle of the outside tire. For a given steering geometry, one can sweep through the possible steering angles of the outside wheel to calculate all possible angles of the inside wheel for comparison to ideal. The limiting angle is one that causes the four-bar to reach its lock point, e.g. when the following condition is met.

$$e = R_3 + R_4 \quad (11)$$

After the four-bar reaches its lock point the four-bar becomes unpredictable. The four-bar equations are only used to calculate the position of the wheels in a left hand turn since the results for a right hand turn would be the same.

Once all steering angles are known, the misalignment error in the inside tire is calculated by comparing the inside steer angle, δ_{ideal} , with the actual inside steer angle caused by the four-bar linkage.

$$Percentage\ Error = \left(\left| \frac{\delta_{ideal} - \delta_{actual}}{\delta_{ideal}} \right| \right) \times 100 \quad (12)$$

This calculation is done for a wide range of four-bar set-ups and each four-bar is rotated to the point of lock.

Fig. 5 shows a representation of the ideal steer of the inside wheel and the actual steering of the inside wheel due to a particular four-bar set-up. The solid black line represents the ideal steering angle for the inside wheel, for a given radius. The dotted line represents a four-bar steering angle for a given radius. One can see that these lines are not on top of each other.

One means of comparing ideal versus actual steering angles is to compute a metric that quantifies the difference between these two curves. The area between the two curves is one such metric, and is easily calculated by using the trapezoidal rule,

$$A_{total} = \sum_i \left(\frac{|\delta_{ideal_1} - \delta_{inside_1}| + |\delta_{ideal_2} - \delta_{inside_2}|}{2} \right) (|R_{outside_1} - R_{outside_2}|). \quad (13)$$

Thus, less total area corresponds to improved accuracy of the four-bar.

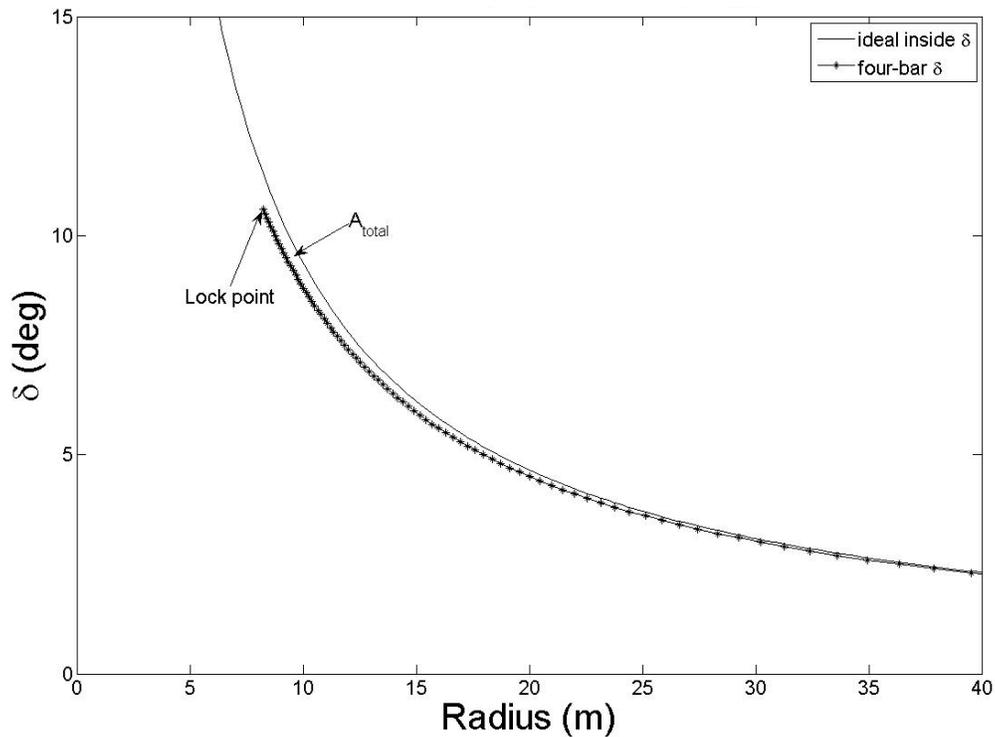


Figure 5. Ideal and four-bar steering of inside wheel.

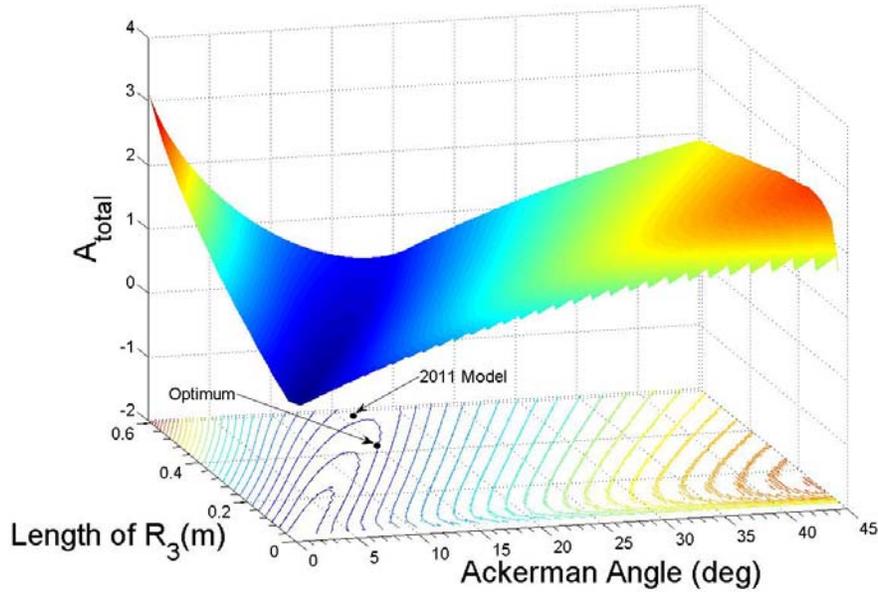


Figure 6. A_{total} associated with four-bar design

Fig. 6 shows the accuracy of different four-bar set-ups with relations to perfect Ackermann steering for a track width of 0.6255 meters and vehicle length of 1.6 meters. The surface plot shows that there is a relationship between R_3 and the AA that gives improved steering, and the plot also shows that there is no trapezoidal steering geometry that gives an ideal geometry over all turn radii.

Considering the best-case geometry for each length of R_3 , one obtains the plot as shown in Fig. 7. Also seen in this figure is the linear fit, showing that the relationship between R_3 and the ideal Ackermann Angle of the steering arm is approximately, but not quite, linear.

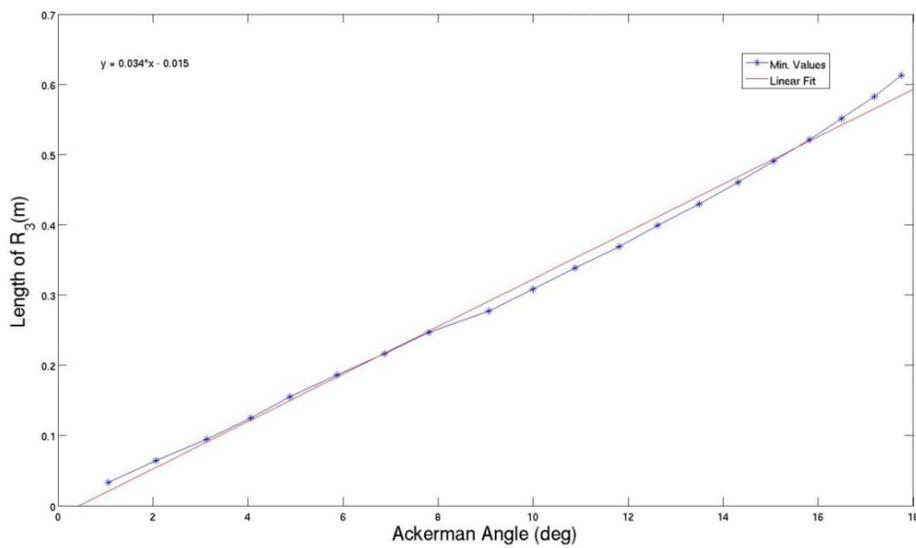


Figure 7. Minimum value of A_{total} represents valley from Fig. 6 for each four-bar setup.

One common method to establish the trapezoidal geometry in practice is to project an imaginary line from the steering arms to the center of the rear axle, and then to use a trapezoidal configuration to connect these arms. If one uses this rule of thumb, one obtains an AA angle of 11.05°. Looking at Fig. 7 we see that the ideal four-bar setup for this rule of thumb only occurs when the steering arm length is roughly 0.3 meters. For all other configurations, this design rule is not ideal.

5. Optimization of Steering Geometry for a Particular Driving Course

The previous error analysis considered a cumulative error that weights all steering angles equally. However, vehicles in competition are not required to have extremely tight turning geometries nor encounter all radii turns with equal frequency. A more appropriate approach would be to weight the errors of common turn geometries most heavily, and to give little or no weight to turns that are rarely if ever encountered.

The Shell Eco-marathon Americas track at the Discovery Green Park has multiple turns of different radii and lengths. Using satellite images from Google maps, the preferred and most common path around the track was traced. The radius and arc length of each turn is then calculated, and the total length of one circuit was also calculated. The pie graph in Fig. 8 illustrates the weighted importance of each particular turn, as determined by the total arc lengths of each radii turn. Because the competition vehicle travels at a nearly constant velocity, even around turns, the following histogram is also representative of the cumulative time the vehicle spends within each turn.

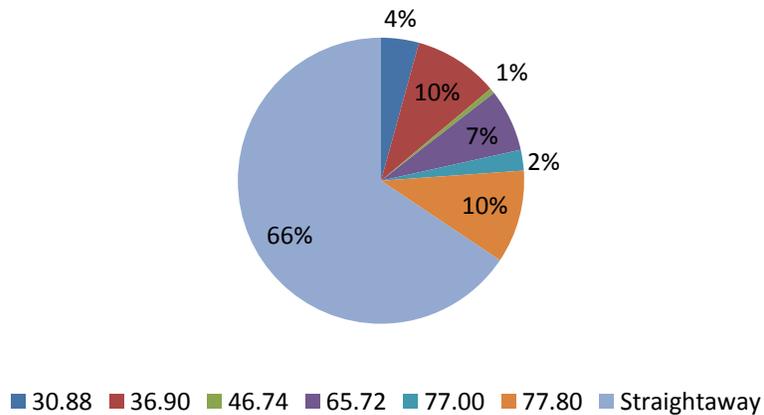


Figure 8. Weighted relationship of distance in straightaway and radii (m) of turns.

Using the pie graph and modeled four-bar analysis, we can then re-weight the results shown previously by the factors given in Fig. 6. Specifically, the steering differences for the four-bar and ideal steer angle of each turn are multiplied by the corresponding weight.

$$E_{total} = \sum_i w_i (|\delta_{ideal} - \delta_{actual}|)_i \quad (14)$$

Each four-bar was then re-evaluated, with the four-bar that acquires the minimum error being the optimal four-bar choice. These results are illustrated in Fig. 9.

The graph shows the amount of error involved with each four-bar. The majority of the turns on the track have a radius greater than 45 meters, and thus there is a large range of four-bars that could possibly be used with relatively small differences in resulting steering geometry.

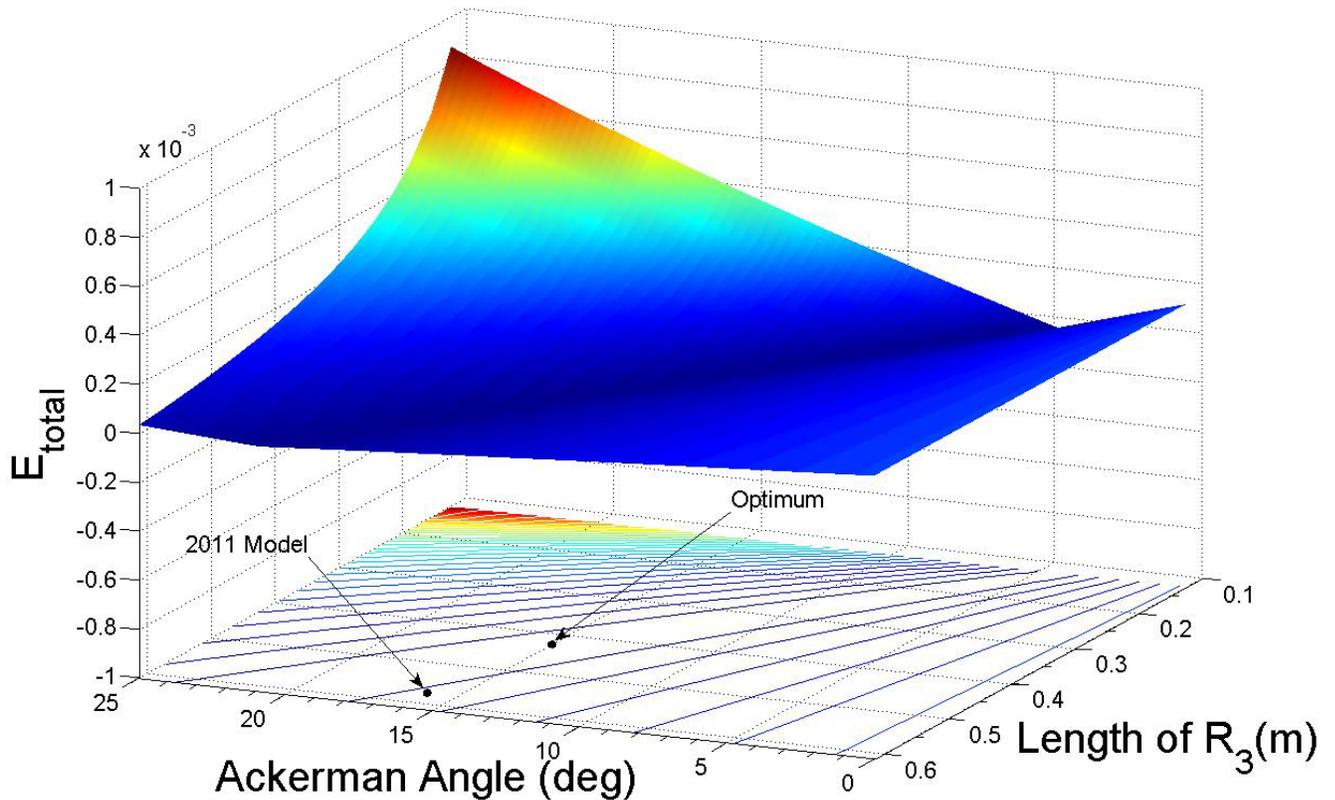


Figure 9. Four-bar design based on track data of Discovery Green in Houston, Texas.

6. Efficiency Losses Due to Incorrect Ackermann Geometry via a Tire Model Analysis

When the tire is not aligned with the direction of travel, it will produce a lateral and longitudinal force. The angle of misalignment between the tire plane and the direction of travel under the tire is called the tire's slip angle. [4] For small angles such as due to tire misalignment, the lateral force can be found with the following relationship, [4] when cornering stiffness (C_α) is a tire property and α is the slip angle [4]:

$$F_y = C_\alpha \alpha. \quad (15)$$

The term, C_α , is called the tire's cornering stiffness and is mostly dependent on the inflation pressure and the normal load on the tire. [4]

To understand the impact of steering misalignment, the following analysis considered the competition entry from the Penn State team in 2010. The total weight of the car and driver for Penn State was 117.02 kg, producing about 1.148 kN of force. Assuming the weight to be evenly distributed across all wheels results in a normal load of 0.3826 kN on each wheel. These tires were operated at a value of 5.17 bar. Using Pacejka’s “magic formula” a value of cornering stiffness can be calculated, [5]

$$C_{\alpha}[N/^\circ] = (a_{30} + a_{31}P) \sin\left(2 \tan^{-1}\left(\frac{F_z}{a_{40} + a_{41}P}\right)\right). \quad (16)$$

In order to calculate C_{α} , Michelin provided the Pacejka model coefficients for radial ply 45-75R16 in as shown Table 1, tires that were used in competition.

a_{30}	a_{31}	a_{40}	a_{41}
57.806	15.101	-0.082	0.186

Table 1. Cornering Stiffness Coefficients for Michelin radial ply 45-75R16.

It is recommended that the coefficients only be used if the operating conditions are in the following limits: $0.3 < F_z < 0.6$ kN and $4 < P < 6$ bar, which the competition vehicle followed. [6] The cornering stiffness was calculated to be 5696 N/rad.

The cornering stiffness, C_{α} , measurement enables a fuel mileage analysis of the Ackermann misalignment. From previous Penn State power measurements, it was determined that the vehicle overcomes a steady drag force of 17.15N averaged over each lap in order to maintain a constant speed of 24.14 kph. From the tire model mentioned before, one can calculate the lateral force involved in cornering by considering the elapsed time in each part of the track. Because the vehicle was moving at a constant speed, the weights that were previously calculated can be used.

For each turn, the misalignment errors in each turn were calculated based on the specific four-bar configuration of each turn. Equation (17) yields the decrease in fuel mileage per run.

$$M_{lost} = M_{actual} \frac{\sum_i w_i \alpha_i C_{\alpha}}{F_{drag\ total}} \quad (17)$$

Fig. 10 shows the loss in fuel mileage relative to the measurements obtained at competition, due to a particular four-bar arrangement. One can see the importance of four-bar geometry and how tire misalignment affects fuel mileage. Choosing the optimal four-bar from Fig. 10, the vehicle would theoretically lose 0.1771 MPG. For the actual geometry used in the competition vehicle and the measured fuel economy of 4700 MPG, the loss due to the current four-bar set-up, 50.0 MPG, only penalizes the vehicle 1.1%. While this seems small, one must recognize that past competition entries have lost ranking based on smaller amounts, which can be particularly frustrating given that steering geometry is easily adjusted to the ideal value with no vehicle redesign.

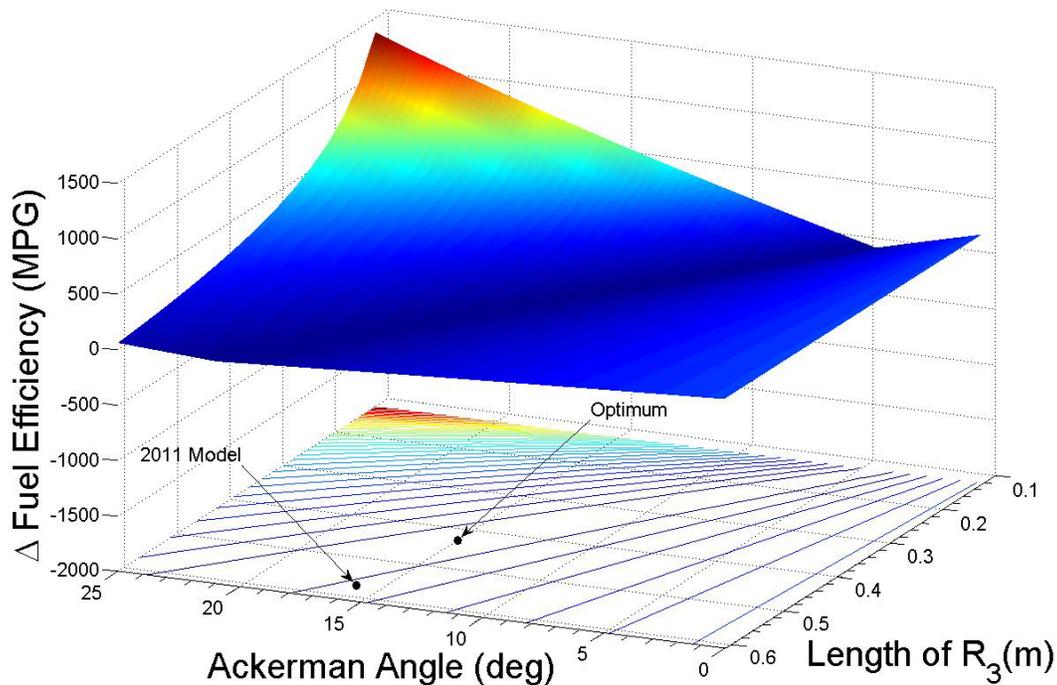


Figure 10. Loss in fuel efficiency based on the four-bar geometry

5. Conclusions

This paper considered the fuel losses associated with the misalignment of the front steering geometry due to the typical trapezoidal Ackermann geometry setup. Starting with a standard front steering geometry, an algorithm was developed to calculate the optimal four-bar geometry for a vehicle and it is seen that such geometries can approach, but never reach, perfect Ackermann geometry on all individual tires simultaneously for low-speed turns. Using turning data collected from prior competitions, predicted performance improvements were given for Penn State’s super-high-mileage entry in the Shell Eco-marathon Americas.

A result of this analysis is the realization that the only way to have perfect Ackermann is to steer each wheel independently, which may be possible by using a servo motor. This could be a costly endeavor and more than likely result in extra weight added to the vehicle with more energy consumed to initially get the vehicle up to speed and to maintain the vehicle at a steady speed. This perpetual tradeoff between vehicle weight versus performance is the central problem of ultra-high-efficiency vehicle challenges.

Additionally, this analysis ignores the weight transfer would affect how the tire tracks along a given radius. Future work could consider this effect by specifically examining the cornering coefficients of the tires and correcting for this accordingly. Another interesting analysis would be to compare the fuel economy losses for higher-speed vehicles, ones where low-speed approximations are not necessarily valid. While faster speeds will cause larger power losses for a given misalignment, the high speeds also generally involve much smaller turn radii. Thus, it is unclear whether a 1.1% improvement in fuel economy could be expected through optimized steering geometries.

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